PERTH MODERN SCHOOL
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Mini Test Chap 10,12,13 & 14

Semester Two 2018
Mathematics Methods
Calc Assumed
(Formula sheet allowed)

Name: Solutions

Time: 35 minutes **Total**: /~~38~~ 33 marks

Working needs to be shown for full marks

Question 1 [1 marks]

In how many ways can a hand of five cards be dealt from a deck of 54 cards?

$^{54}C_5 = 3\,162\,510$

Question 2 [2 marks]

A five-letter 'word' is to be made by arranging the letters of the word WHOLEGRAIN. What is the probability that the word begins with a vowel?

$$\frac{4\times9\times8\times7\times6}{10\times9\times8\times7\times6} = 0.4$$

Question 3 [4 marks]

A two-digit number is to be formed from the set of numbers {1, 2, 3, 4, 5, 6}. No repetition is allowed. Find the probability that the number:

a is even $\frac{3\times5}{6\times5} = \frac{1}{2}$

b is less than 30 $\frac{1\times5}{6\times5} + \frac{1\times5}{6\times5} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$.

c is even given that it is less than 30. $P(E|<30)$

$$= \frac{P(E \cap <30)}{P(<30}$$

$$= \frac{\frac{1}{2}\times\frac{1}{3}}{\frac{1}{3}} = \frac{1}{2}.$$

Question 4 [1 marks]

The period of the graph of $y = 3\sin(\frac{1}{2}x - \pi) + 4$ is

$P = \frac{2\pi}{\frac{1}{2}}$

$= 4\pi$ ✓

Question 5 [3 marks]

A graph of the function with equation $y = \cos x$ is transformed by a dilation of factor 2 from the y-axis, and a translation of $\frac{\pi}{3}$ units in the negative direction of the x-axis and 1 unit in the negative direction of the y-axis. What is the new equation?

$y = \cos\left(\frac{1}{2}\left(x + \frac{\pi}{3}\right)\right) - 1$ ✓ ✓ ✓

Question 6 [2 marks]

For the equation $\cos(2x) = 1$, the sum of the solutions in the interval $[0, 2\pi]$

Calculator

Solve$(\cos(2x) = 1 \mid 0 \leq x \leq 2\pi)$

$x = 0, 2\pi, \pi$ ✓

$\therefore$ Sum $= 3\pi$ ✓

No Calculator

$\cos 2x = 1 \quad [0, 2\pi]$

$\theta = 2x \quad [0, 4\pi]$

$\cos\theta = 1$

$\theta = 0, 2\pi, 4\pi$

$\therefore \quad x = 0, \pi, 2\pi$

Question 7 [2 marks]

If $\sin\alpha = 0.8$ and $\cos\alpha = 0.6$, what is the value of $\sin(\frac{\pi}{2} + \alpha)$.

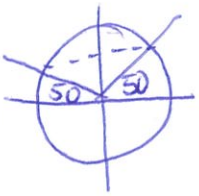
$\sin\left(\frac{\pi}{2} + \alpha\right) = 0.6$ ✓

Because - on formula sheet

$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

$\sin 50° = \sin(180 - 50)$

$\sin\theta = \sin(\pi - \theta)$



Question 8 [1 marks]

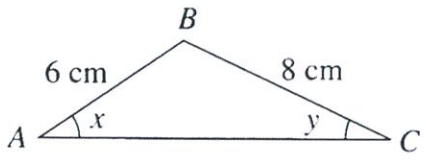
The vertical distance above the ground of a point on a wheel as it rotates is given by $D(t) = 2 - 2\sin(3\pi t)$, where t is the time in seconds. What is the time in seconds for a full rotation of the wheel.

$\frac{2}{3}$

2.

Question 9 [4 marks]

In triangle ABC as shown, $\sin x = \frac{3}{7}$.



What is the value of sin y.

$\frac{\sin y}{6} = \frac{\sin x}{8}$ ✓

$\sin y = \frac{3}{7} \times \frac{1}{8} \times 6$

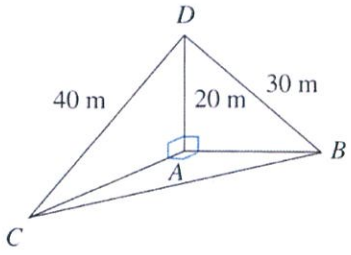
$= \frac{18}{56}$

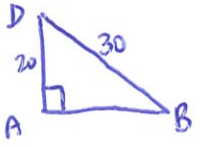
$= \frac{9}{28}$ ✓

Show work for full marks.

Question 10 [4 marks]

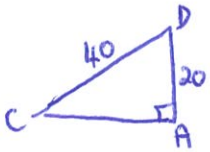
A vertical mast, AD, of height 20 m is supported by two cables attached to the ground at C and B as shown in the diagram. $\angle CAB$ is a right angle. Cable CD is of length 40 m and cable BD is of length 30 m.





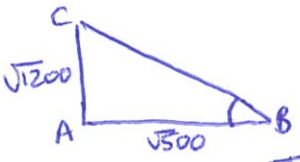
$30^2 = 20^2 + AB^2$

$AB = \sqrt{500}$ ✓



$40^2 = 20^2 + CA^2$

$CA = \sqrt{1200}$ ✓



$\tan \angle ABC = \frac{\sqrt{1200}}{\sqrt{500}}$ ✓

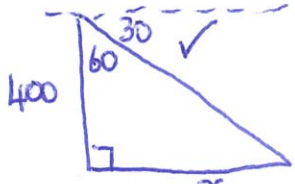
$\approx 57°$ ✓

What is the angle ABC, to the nearest degree?

Show work for full marks.

Question 11 [3 marks]

From a point on a cliff 400 m above sea level, the angle of depression to a boat is 30°. Find the distance from the foot of the cliff to the boat.



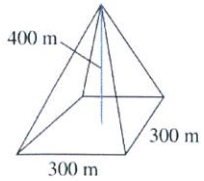
$\tan 60 = \frac{x}{400}$ ✓

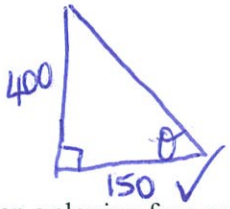
$x = 400 \tan 60$

$\approx 692.8 \text{ m}$ ✓

Question 12 [3 marks]

The diagram shown is a right square-pyramid of height 400 m with its base a square of side 300 m.





$\tan \theta = \frac{400}{150}$ ✓

$= \frac{8}{3}$

If θ is the angle between a sloping face and the base, form an equation which will give the correct value of θ.

or $\tan^{-1}\left(\frac{8}{3}\right)$ ✓

12

4.

Question 13 **[5 marks]**

A highly volatile substance has an initial mass of 1200 g and its mass is reduced by 12% each second.

a Write a formula that gives the mass of the substance (m) at time (t) seconds.

$M = 1200(0.88)^t$ ✓

b Rearrange this formula to make t the subject.

c What mass remains after 10 seconds, correct to 2 decimal places?

334.20 seconds. ✓

d Calculate how long (to the nearest second) it takes until the mass is 10 grams.

37 seconds. ✓

e After how many seconds (to the nearest second) is the mass less than 1 gram?

56 seconds. ✓

Question 14 **[1 marks]**

The equation of the graph shown could be

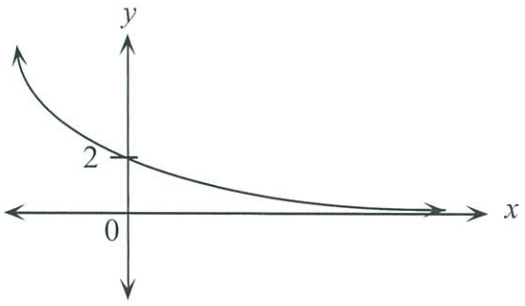
A $y = 2^{-x} + 1$

B $y = 2^{-2x}$

C $y = 2^{-x+2}$

D $y = 2 \times 2^{-x+2}$

E $y = 2 \times 2^{-x}$



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